

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MEI STRUCTURED MATHEMATICS

4761

Mechanics 1

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

Section A (36 marks)

- 1 A particle is thrown vertically upwards and returns to its point of projection after 6 seconds. Air resistance is negligible.

Calculate the speed of projection of the particle and also the maximum height it reaches. [4]

- 2 Force \mathbf{F}_1 is $\begin{pmatrix} -6 \\ 13 \end{pmatrix}$ N and force \mathbf{F}_2 is $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ N, where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are vectors east and north respectively.

(i) Calculate the magnitude of \mathbf{F}_1 , correct to three significant figures. [2]

(ii) Calculate the direction of the force $\mathbf{F}_1 - \mathbf{F}_2$ as a bearing. [3]

Force \mathbf{F}_2 is the resultant of all the forces acting on an object of mass 5 kg.

(iii) Calculate the acceleration of the object and the change in its velocity after 10 seconds. [3]

- 3 A train consists of an engine of mass 10 000 kg pulling one truck of mass 4000 kg. The coupling between the engine and the truck is light and parallel to the track.

The train is accelerating at 0.25 m s^{-2} along a straight, level track.

(i) What is the resultant force on the train in the direction of its motion? [2]

The driving force of the engine is 4000 N.

(ii) What is the resistance to the motion of the train? [1]

(iii) If the tension in the coupling is 1150 N, what is the resistance to the motion of the truck? [2]

With the same overall resistance to motion, the train now climbs a uniform slope inclined at 3° to the horizontal with the same acceleration of 0.25 m s^{-2} .

(iv) What extra driving force is being applied? [3]

- 4 Fig. 4 shows the unit vectors \mathbf{i} and \mathbf{j} in the directions of the cartesian axes Ox and Oy , respectively. O is the origin of the axes and of position vectors.

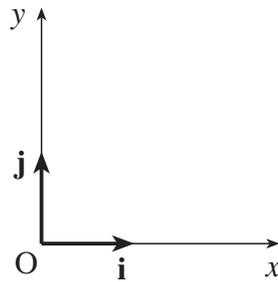


Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geq 0$, where t is time.

- (i) Show that the path of the particle cuts the x -axis just once. [2]

- (ii) Find an expression for the velocity of the particle at time t .

Deduce that the particle never travels in the \mathbf{j} direction. [3]

- (iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]

- 5 You should neglect air resistance in this question.

A small stone is projected from ground level. The maximum height of the stone above horizontal ground is 22.5 m.

- (i) Show that the vertical component of the initial velocity of the stone is 21 m s^{-1} . [2]

The speed of projection is 28 m s^{-1} .

- (ii) Find the angle of projection of the stone. [2]

- (iii) Find the horizontal range of the stone. [4]

Section B (36 marks)

- 6 A toy car is travelling in a straight horizontal line.

One model of the motion for $0 \leq t \leq 8$, where t is the time in seconds, is shown in the velocity–time graph Fig. 6.

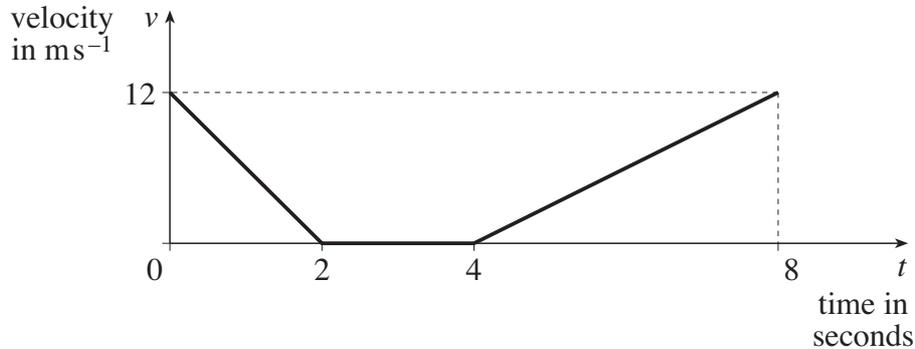


Fig. 6

- (i) Calculate the distance travelled by the car from $t = 0$ to $t = 8$. [2]
- (ii) How much less time would the car have taken to travel this distance if it had maintained its initial speed throughout? [1]
- (iii) What is the acceleration of the car when $t = 1$? [2]

From $t = 8$ to $t = 14$, the car travels 58.5 m with a new constant acceleration, $a \text{ m s}^{-2}$.

- (iv) Find a . [2]

A second model for the velocity, $v \text{ m s}^{-1}$, of the toy car is

$$v = 12 - 10t + \frac{9}{4}t^2 - \frac{1}{8}t^3, \text{ for } 0 \leq t \leq 8.$$

This model agrees with the values for v given in Fig. 6 for $t = 0, 2, 4$ and 6. [Note that you are not required to verify this.] Use this second model to answer the following questions.

- (v) Calculate the acceleration of the car when $t = 1$. [3]
- (vi) Initially the car is at A. Find an expression in terms of t for the displacement of the car from A after the first t seconds of its motion.
Hence find the displacement of the car from A when $t = 8$. [5]
- (vii) Explain with a reason what this model predicts for the motion of the car between $t = 2$ and $t = 4$. [3]

- 7 A box of weight 147 N is held by light strings AB and BC. As shown in Fig. 7.1, AB is inclined at α to the horizontal and is fixed at A; BC is held at C. The box is in equilibrium with BC horizontal and α such that $\sin \alpha = 0.6$ and $\cos \alpha = 0.8$.

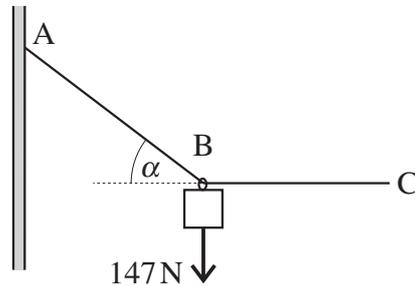


Fig. 7.1

- (i) Calculate the tension in string AB. [3]
- (ii) Show that the tension in string BC is 196 N. [2]

As shown in Fig. 7.2, a box of weight 90 N is now attached at C and another light string CD is held at D so that the system is in equilibrium with BC still horizontal. CD is inclined at β to the horizontal.

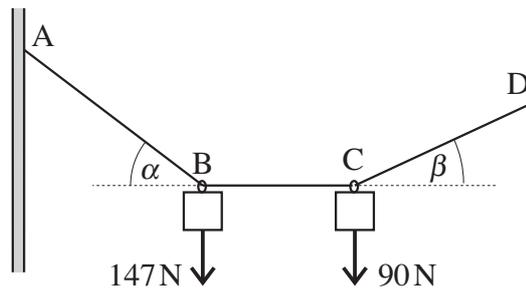


Fig. 7.2

- (iii) Explain why the tension in the string BC is still 196 N. [2]
- (iv) Draw a diagram showing the forces acting on the box at C.

Find the angle β and show that the tension in CD is 216 N, correct to three significant figures. [7]

The string section CD is now taken over a smooth pulley and attached to a block of mass M kg on a rough slope inclined at 40° to the horizontal. As shown in Fig. 7.3, the part of the string attached to the box is still at β to the horizontal and the part attached to the block is parallel to the slope. The system is in equilibrium with a frictional force of 20 N acting on the block **up** the slope.

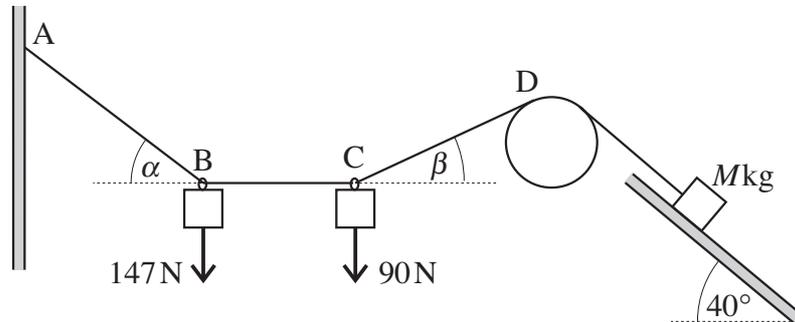


Fig. 7.3

(v) Calculate the value of M .

[4]

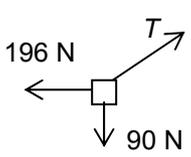
Mark Scheme 4761
June 2006

Q 1	mark	Sub
$0 = u - 9.8 \times 3$ $u = 29.4$ so 29.4 m s^{-1} $s = 0.5 \times 9.8 \times 9 = 44.1$ so 44.1 m	M1 <i>uvast</i> leading to <i>u</i> with $t = 3$ or $t = 6$ A1 Signs consistent M1 <i>uvast</i> leading to <i>s</i> with $t = 3$ or $t = 6$ or their <i>u</i> F1 FT their <i>u</i> if used with $t = 3$. Signs consistent. Award for 44.1, 132.3 or 176.4 seen. [Award maximum of 3 if one answer wrong]	4 4
Q 2	mark	Sub
(i)	$\sqrt{(-6)^2 + 13^2} = 14.31782\dots$ so 14.3 N (3 s. f.)	2
(ii)	Resultant is $\begin{pmatrix} -6 \\ 13 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$ Require $270 + \arctan \frac{8}{3}$ so $339.4439\dots^\circ$ so 339°	3
(iii)	$\begin{pmatrix} -3 \\ 5 \end{pmatrix} = 5\mathbf{a}$ so $(-0.6\mathbf{i} + \mathbf{j}) \text{ m s}^{-2}$ change in velocity is $(-6\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$	3 8

Q 3	mark	Sub
(i) $F = 14000 \times 0.25$ so 3500 N	M1 A1	2
(ii) $4000 - R = 3500$ so 500 N	B1	1
(iii) $1150 - R_T = 4000 \times 0.25$ so 150 N	M1 A1	2
(iv) either Component of weight down slope is Extra driving force is cpt of mg down slope $14000g \sin 3^\circ$ $= 14000 \times 9.8 \times 0.0523359... = 7180.49...$ so 7180 N (3 s. f.) or $D - 500 - 14000g \sin 3 = 14000 \times 0.25$ $D = 11180.49...$ so extra is 7180 N (3 s. f.)	M1 M1 A1 M1 M1 A1	3 8

Q 4	mark	Sub
(i) either Need j cpt 0 so $18t^2 - 1 = 0$ $\Rightarrow t^2 = \frac{1}{18}$. Only one root as $t > 0$	M1 Need not solve E1 Must establish only one of the two roots is valid	
or Establish sign change in j cpt Establish only one root	B1 B1	2
(ii) $\mathbf{v} = 3 \mathbf{i} + 36t \mathbf{j}$ Need i cpt 0 and this never happens	M1 Differentiate. Allow i or j omitted A1 E1 Clear explanation. Accept 'i cpt always there' or equiv	3
(iii) $x = 3t$ and $y = 18t^2 - 1$ Eliminate t to give $y = 18\left(\frac{x}{3}\right)^2 - 1$ so $y = 2x^2 - 1$	B1 Award for these two expressions seen. M1 t properly eliminated. Accept any form and brackets missing A1 cao	3 8
Q 5	mark	Sub
(i) $0^2 = V^2 - 2 \times 9.8 \times 22.5$ $V = 21$ so 21 m s^{-1}	M1 Use of appropriate <i>uvast</i> . Give for correct expression E1 Clearly shown. Do not allow $v^2 = 0 + 2gs$ without explanation. Accept using $V = 21$ to show $s = 22.5$.	2
(ii) $28 \sin \theta = 21$ so $\theta = 48.59037\dots$	M1 Attempt to find angle of projection. Allow $\sin \leftrightarrow \cos$. A1	2
(iii) Time to highest point is $\frac{21}{9.8} = \frac{15}{7}$ Distance is $2 \times \frac{15}{7} \times 28 \times \cos(\text{their } \theta)$.. 79.3725... so 79.4 m (3 s. f.)	B1 Or equivalent (time of whole flight) M1 Valid method for horizontal distance. Accept $\frac{1}{2}$ time. Do not accept 28 used for horizontal speed or vertical speed when calculating time. B1 Horizontal speed correct A1 cao. Accept answers rounding to 79 or 80. [If angle with vertical found in (ii) allow up to full marks in (iii). If $\sin \leftrightarrow \cos$ allow up to B1 B1 M0 A1] [If $u^2 \sin 2\theta / g$ used then M1* Correct formula used. FT their angle. M1 Dep on *. Correct subst. FT their angle. A2 cao]	4 8

Q 6	mark	Sub
(i) $0.5 \times 2 \times 12 + 0.5 \times 4 \times 12$ so 36 m	M1 Attempt at sum of areas or equivalent. No extra areas. A1	2
(ii) $8 - \frac{36}{12} = 5$ seconds	B1 cao	1
(iii) -6 m s^{-2}	M1 Attempt at accn for $0 \leq t \leq 2$ B1 must be - ve or equivalent	2
(iv) $58.5 = 12 \times 6 + 0.5 \times a \times 36$ so $a = -0.75$	M1 Use of <i>uvast</i> with 12 and 58.5 A1	2
(v) $a = -10 + \frac{9}{2}t - \frac{3}{8}t^2$ $a(1) = -10 + \frac{9}{2} - \frac{3}{8} = -5.875$	M1 Differentiation A1 A1 cao	3
(vi) $s = \int \left(12 - 10t + \frac{9}{4}t^2 - \frac{1}{8}t^3 \right) dt$ $= 12t - 5t^2 + \frac{3}{4}t^3 - \frac{1}{32}t^4 + C$ $s = 0$ when $t = 0$ so $C = 0$ $s(8) = 32$	M1 Attempt to integrate A1 At least one term correct A1 All correct. Accept + <i>C</i> omitted A1* Clearly shown A1 cao (award even if A1* is not given)	5
(vii) either $s(2) = 9.5$ and $s(4) = 8$ Displacement is negative Car going backwards or Evaluate $v(t)$ where $2 < t < 4$ or appeal to shape of the graph Velocity is negative Car going backwards	B1 Both calculated correctly from their s . No further marks if their $s(2) \leq s(4)$ E1 E1 Do <i>not</i> need car going backwards <i>throughout</i> the interval. B1 e.g. $v(3) = -1.125$ No further marks if their $v \geq 0$ E1 E1 Do <i>not</i> need car going backwards <i>throughout</i> the interval [Award WW2 for 'car going backwards'; WW1 for velocity or displacement negative]	3

Q 7	mark	Sub
(i) $T_{AB} \sin \alpha = 147$ so $T_{AB} = \frac{147}{0.6}$ $= 245$ so 245 N	M1 Attempt at resolving. Accept $\sin \leftrightarrow \cos$. Must have T resolved and equated to 147. B1 Use of 0.6. Accept correct subst for angle in wrong expression. A1 Only accept answers agreeing to 3 s. f. [Lami: M1 pair of ratios attempted; B1 correct sub; A1]	3
(ii) $T_{BC} = 245 \cos \alpha$ $= 245 \times 0.8 = 196$	M1 Attempt to resolve 245 and equate to T , or equiv Accept $\sin \leftrightarrow \cos$ E1 Substitution of 0.8 clearly shown [SC1 $245 \times 0.8 = 196$] [Lami: M1 pair of ratios attempted; E1]	2
(iii) Geometry of A, B and C and weight of B the same and these determine the tension	E1 Mention of two of: same weight: same direction AB: same direction BC E1 Specific mention of same geometry & weight or recognition of same force diagram	2
(iv) 	No extra forces. B1 Correct orientation and arrows B1 'T' 196 and 90 labelled. Accept 'tension' written out.	
either Realise that 196 N and 90 N are horiz and vert forces where resultant has magnitude and line of action of the tension $\tan \beta = 90/196$ $\beta = 24.6638\dots$ so 24.7 (3 s. f.) $T = \sqrt{196^2 + 90^2}$ $T = 215.675\dots$ so 216 N (3 s. f.) or $\uparrow T \sin \beta - 90 = 0$ $\rightarrow T \cos \beta - 196 = 0$ Solving $\tan \beta = \frac{90}{196} = 0.45918\dots$ $\beta = 24.6638\dots$ so 24.7 (3 s. f.) $T = 215.675\dots$ so 216 N (3 s. f.)	M1 Allow for only β or T attempted B1 Use of $\arctan(196/90)$ or $\arctan(90/196)$ or equiv A1 M1 Use of Pythagoras E1 B1 Allow if $T = 216$ assumed B1 Allow if $T = 216$ assumed M1 Eliminating T , or... A1 [If $T = 216$ assumed, B1 for β ; B1 for check in 2 nd equation; E0]	7
(v) Tension on block is 215.675.. N (pulley is smooth and string is light) $M \times 9.8 \times \sin 40 = 215.675\dots + 20$ $M = 37.4128\dots$ so 37.4 (3 s. f.)	B1 May be implied. Reasons not required. M1 <i>Equating</i> their tension on the block unresolved ± 20 to weight component. If equation in any other direction, normal reaction must be present. A1 Correct A1 Accept answers rounding to 37 and 38	4

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General Comments

Most candidates found something that they could do and quite a few could do most of the paper but there were several widespread difficulties. First there was a part question that very few could do: Q 7 (iii) proved inaccessible to most candidates, partly because the response we were looking for was too sophisticated and partly because many of the candidates were not able to express their (possibly correct) ideas clearly. Secondly, many candidates did not realise that Q 2 (iii) was a question about vectors. Thirdly, the techniques required for Q 4 were not widely known. Account was taken of these problems when setting grade thresholds at the Award.

There were also some quite marked differences to the responses of candidates in recent sessions.

There were rather more very low scores.

Quite a few candidates showed low levels of ability when dealing with arithmetical and algebraic expressions. For example: $21\cos\theta = 28$ gives $\theta = \frac{28}{21\cos}$ (which is then evaluated as $\frac{28}{\cos 21}$):

$\sqrt{(-6)^2 + 13^2}$ written and evaluated as $\sqrt{-6^2 + 13^2}$.

There seemed to be more scripts that were poorly presented (for instance: parts of a question unlabelled and mixed in together: 3° written badly in the working and then misread as 30).

There were more examples of attempts at the projectile problem based on the assumption that the position of a projectile could be determined by assuming that the motion was in the straight line defined by the initial velocity.

There were more examples of constant acceleration problems worked (at least in part) as if they were zero acceleration problems by using $distance = speed \times time$.

The ability of candidates to answer parts of the questions sometimes seemed to depend more on the centre than on the ability of the candidates, as measured by responses to the other questions.

There were, of course, many scripts that were beautifully presented with clear, logical working and accuracy shown throughout and which demonstrated a good understanding of the principles and techniques used in this component.

Comments on Individual Questions

Section A

1 Kinematics of a particle moving in a vertical line

This rather standard question was done quite well by many candidates except that there was widespread confusion about when to use the time of flight and when to use half of this. Many candidates used the same time for the half and the full motion. There seemed to be fewer sign errors than seen in similar problems in recent sessions.

2 Magnitude of a vector, the difference of two vectors, the direction of a vector as a bearing and some simple dynamics and kinematics using vectors

- (i) Almost every candidate found the magnitude of the vector correctly.

- (ii) Most candidates found the difference of the two vectors correctly. Most then correctly used a trigonometric ratio but could not generate the required bearing from their angle. A clear diagram would have helped most of those that got it wrong.
- (iii) Few candidates realised that this was a vector question and so gave only scalar answers. Typically, whole centres set about this the right way with some of the candidates from such a centre scoring marks on this part while struggling elsewhere.

3 Newton's second law applied to a train, tension in a coupling and movement on a slope

- (i) Almost all of the candidates realised that they should apply Newton's second law. A surprising number applied it only to the engine. A number of candidates think that the law is $F - mg = ma$, presumably from applying it to vertical motion.
- (ii) Most candidates produced an answer consistent with their answer to part (i). Quite a few wasted time by starting the problem again from scratch instead of arguing that $4000 - R = 3500$.
- (iii) Almost everyone who knew Newton's second law correctly managed to answer this question with relatively few wrongly applying the driving force to the truck – a considerable improvement on the answering of similar problems in recent sessions.
- (iv) Many candidates failed to make much progress with this part. The most common reason was the inability of candidates to write down an expression for the component of the weight down the slope; many omitted g and others used cosine instead of sine. Quite a few candidates misread their own (untidy) writing and turned 3° into 30 . Those who realised that the extra force had the same magnitude as the component of weight down the slope were usually successful although some wrongly added in the resistance term. Those who applied Newton's second law again did not usually fare so well. There were many sign errors and missing terms and many candidates took the resistance to be $(500 + 150)$ N. I am not aware of any candidate who applied Newton's second law in any direction but that of the slope who obtained the correct answer.

4 Motion of a particle in a plane in vector form and the Cartesian equation of its path

It seemed that many candidates were not familiar with much of the content examined in this question.

- (i) Only a few candidates tackled this with confidence and skill. Many thought that the \mathbf{i} component should be zero; many came to this conclusion by the argument that they wanted the \mathbf{j} component zero so they substituted $\mathbf{j} = \mathbf{0}$ giving $\mathbf{r} = 3t\mathbf{i}$. Those who worked with $18t^2 - 1 = 0$ often either found only one root or failed to explain why the negative root could be ignored. Some candidates showed a poor grasp of the nature of vectors by trying to solve $3t + 18t^2 - 1 = 0$.

Those who tried to show that the path crossed the x axis by substituting values for t , usually did not make it clear that they were looking for a sign change in the \mathbf{j} component.

- (ii) Most candidates correctly differentiated but many lost the **i** or the **j** or both in the process. Unlike similar questions in recent sessions, the candidates were told to find a velocity before asking them a question about the direction of motion and so most were trying to do the right thing. Many clearly stated that for motion in the **j** direction the **i** component must be zero but this is not possible as it is constant at 3: others were not so clear and the answers were interpreted generously.
- (iii) The minority of the candidates who knew what to do, mostly did it well. The usual errors were to omit the brackets and so substitute $\frac{x^2}{3}$ instead of $\frac{x^2}{9}$ or fail to simplify the term from $\frac{18}{9}$ to 2.

5 A projectile question that works from maximum height and speed of projection to angle of projection and horizontal range

Many candidates scored well on this question and there seemed to be fewer errors with signs than in recent sessions. However, many candidates used a sequence of formulae when one would have done and used sledgehammer methods on simple equations, for instance using the quadratic formula to solve a quadratic equation with no constant term; one can see from the scripts how poor technique of this type takes up a lot of the time allocation and how the extra working has increased the chance of errors.

- (i) Many candidates obtained the given result with a satisfactory method. Candidates who take $u = 0$ and $g = 9.8$ should explain why this gives v that is the *initial* vertical component of velocity.
- (ii) Most candidates knew what to do. Common mistakes were to confuse sine and cosine and, rather surprisingly, think that the angle of projection was an angle with the upward vertical.
- (iii) A lot of candidates knew what to do and many of them did it well. A common mistake was to find only half the time of flight.

6 Motion in a straight line involving a $v - t$ graph, constant acceleration formulae, calculus and interpretation of a model

Many candidates answered most of this question very well. The most common errors from strong candidates were found in their interpretation in part (vii) and omission of any working showing consideration of an arbitrary constant in part (vi). Many weaker candidates tried to use the constant acceleration formulae inappropriately and some confused differentiation with integration.

- (i) Most candidates answered this part correctly but some included an extra 2×12 term.
- (ii) The only common error here was to give the time it would have taken to travel the distance instead of how much *less* time.
- (iii) Many obtained the correct answer; a positive answer was quite common.

- (iv) Most of the candidates used the *uvast* results and got the right answer; those who tried to use a velocity – time graph approach often confused themselves. A quite large number of candidates used $\text{speed} = \frac{\text{distance}}{\text{time}}$ as part of their calculation.
- (v) This was generally done well but quite a few candidates evaluated $v(1)$ and some integrated.
- (vi) Most candidates knew they should integrate and many of them did so accurately. However, many failed to show any arbitrary constant or failed to show it was zero, some leaving $+ C$ in the expression and then correctly obtaining the 32 with a separate definite integration from 0 to 8.
- (vii) Very many candidates thought that there was zero motion because the $v - t$ graph above showed this and supported their argument by showing $v(2) = v(4) = 0$. Some came to the same conclusion without appealing to the graph above. However, many candidates knew exactly what was required of them and worked on the right lines. Relatively few both supported their arguments by calculations (either showing that the displacement was negative or the velocity was negative between the given times) and also finished by saying that the car was reversing (or similar words).

7

The static equilibrium of connected bodies

For many candidates this proved the hardest question on the paper but others seemed to do well at much of it without too much difficulty. In parts (i) and (ii), many candidates found the angle and then worked out its sine and cosine, often introducing rounding errors.

- (i) Many candidates did not realise that they should resolve vertically. Those who used force triangles were especially prone to resolving the 147.
- (ii) The lack of clarity in some of the working suggested over reliance on the given answer.
- (iii) This was found to be quite the most difficult question to answer correctly on the paper. Very few candidates gave a clear reason and fewer a complete reason. A common statement seen was that ‘the forces were still in equilibrium’ without any reference to attributes of the system that had not changed.
- (iv) The diagrams were often sound but many had wrong labelling (e. g. both strings marked with T) or (some) arrows missing. Many candidates gave up at this point. However, many continued and spotted how the tension must be related to the other forces acting on the system; others correctly resolved horizontally and vertically and then solved. Others found just one equation and substituted the given value of T , deducing the angle but not checking in a second equation.
- (v) Well done by many of the candidates, indeed by more than expected. However, quite a few made no progress or simply found the tension in the string (quite often thinking that the 216 had to be resolved). Any equations found were often inaccurate because of sign errors, resolving the tension or using a mass instead of a weight component.